UNIL | Université de Lausanne
HEC Lausanne

An Analytical Framework for Public Debt Management

3rd PDM conference, World Bank.

Jean-Paul Renne

3-4 October 2024

The paper in a nutshell

Objective:

- To propose an analytical framework to assess cost/risk performances of debt issuance strategies.
- Opens the door to optimal issuance strategy design.

Approach:

- Computation of cost & risk measures based on a stochastic macro-fin. model.
- In the model:
  - Government decides on debt maturity and indexation.
  - Default is possible (fiscal limits).
  - Government issuance choices affect bond prices.

Findings:

- Optimal strategies depend on inflation/GDP/surplus dynamics.
- Performances of nominal debt (versus inflation-linked) depends on the predominance of supply/demand shocks in the economy.
- GDP indexation too costly to be part of optimal strategies.

Literature on optimal public debt management

- **Tax smoothing**:
  Optimal to issue bonds linked to government expenditure to smooth taxes over time (Barro, 1979). Barro (1995): substantial moral hazard. Angeletos (2002): potential in conventional bonds to achieve tax smoothing. Faraglia et al. (2010): recommendations from this approach are usually unrealistic and non-robust.
- **Debt indexation**:
  Bohn (1990) and Barro (2003): optimal inflation indexation given tax-smoothing objective. Schmid et al. (2023): issuing ILBs prevents future govts from monetizing debt ex-post. Froot et al. (1989), Shiller (1998), Kamstra and Shiller (2009, 2010), Borensztein and Mauro (2004), Pienkowski (2017): GDP indexation mitigate the adverse effects of negative economic growth.
- **Credit risk**:
  Missale (1997, 2012): fiscal insurance is constrained by the necessity to maintain credibility. Practitioners' simulation approaches abstract from potential sovereign credit risk (e.g., Bergstrom et al., 2002; Pick and Anthony, 2008; Bolder and Deeley, 2011; Balibek and Memis, 2012; Bernaschi et al., 2019).
- **Originality of the present study**: Balance between structural/empirical aspects. Performance metrics capable of integrating both tax-smoothing and debt sustainability objectives (e.g., upper percentiles of the debt-to-GDP distribution).

Paper	Default	Model and optimality criteria	Debt. Instr.
Greenwood et al. (2015)	—	3-period model. Monetary services from holding riskless short-term securities. No inflation. Criterion: Social welfare.	TS-R
Missale and Giavazzi (2005)	—	Simple (i.i.d.) dynamics of inflation, output growth and exchange rate. Criterion: quantiles of debt-to-GDP ratio.	ST-RNX
Debortoli et al. (2017)	—	Stochast. equilib. model with fiscal policy distortions. Govt cannot commit to fiscal policy. Criterion: social welfare.	ST-N, GD-N
Missale and Blanchard (1994)	—	Study the gov temptation to inflate debt away. Loss function including tax rate	ST-N, LT-N
Drudi and Giordano (2000)	✓	3-period model. Criterion: ad-hoc loss function involving tax rate, inflation, and default costs.	3-period model, ST-RNX, LT-RNX
Angeletos (2002)	—	Stochastic production economy with distortionary taxes. Incomplete markets. Criterion: Social welfare.	ST-R and P-R
Buera and Nicolini (2004)	—	Stochastic production economy with distortionary taxes. Incomplete markets. Criterion: Social welfare.	TS-R
Faraglia et al. (2010)	—	Stochastic production economy with distortionary taxes. Incomplete markets. Criterion: Social welfare.	TS-N
Bhandari et al. (2017)	—	Stochastic production economy with distortionary taxes. Criterion: Social welfare	ST-R, P-R
Bigio et al. (2023)	—	Deterministic model. The government faces liquidity costs during bond auctions; the model also features preferred-habitat investors. Criterion: social welfare.	TS-N
de Lannoy et al. (2022)	—	General stoch. macro-fi models. Criterion: Social welfare	TS-R
Bocola and Dovis (2019)	✓	Small-scale macro-finance model *à la* Cole and Kehoe (2000). Decay coupon rate is time-varying (endogenous). Criterion: Social welfare.	GD-N

Overview of modeling ingredients

- **Government issuances**:

  Perpetuities with indexed and geometrically-decaying coupons.

- **Probability of default**:

  >0 when debt-to-GDP larger than fiscal limit.

- **Stochastic discount factor**:

  parametric (Epstein-Zin in application).

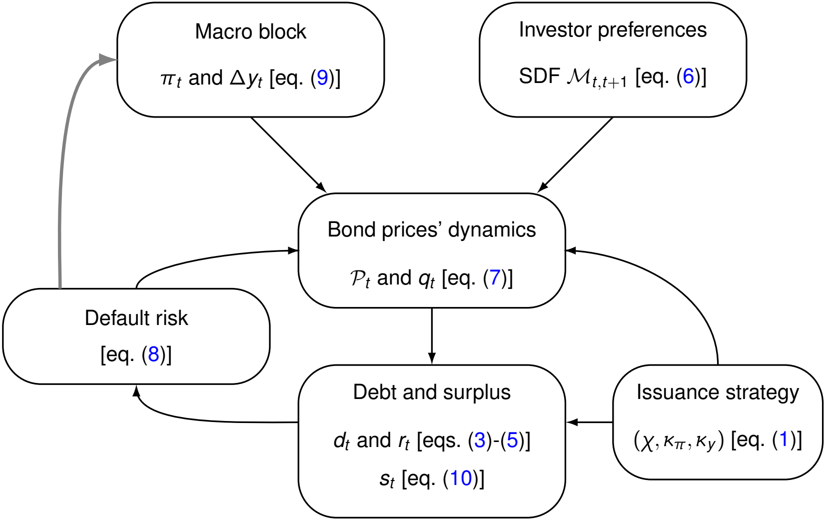
- **Macro dynamics**:

  Inflation, GDP growth, and budget surplus depend on regimes (Markov chain); budget surplus also affected by Gaussian shocks.

- **Feedback mechanism**:

  GDP growth falls upon sovereign default.

Figure: Schematic representation of the model



Perpetuities with indexed and geom.-decaying coupons

- The government issues perpetuities with geometrically-decaying coupons.

⇒ Geometrically-decaying coupons = standard in macro literature (e.g., Leland, 1998; Woodford, 2001; Hatchondo and Martinez, 2009).

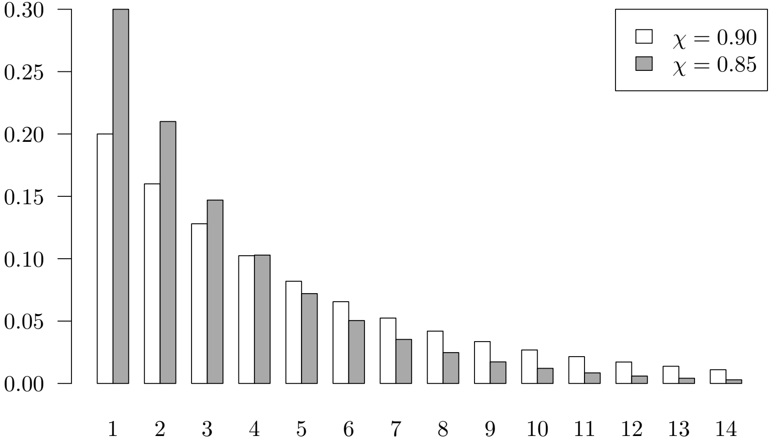
⇒ Present paper: Extension to accommodate indexation (inflation and/or GDP).

- Payoff of perpetuities (on date $t+h$, and in dollars):

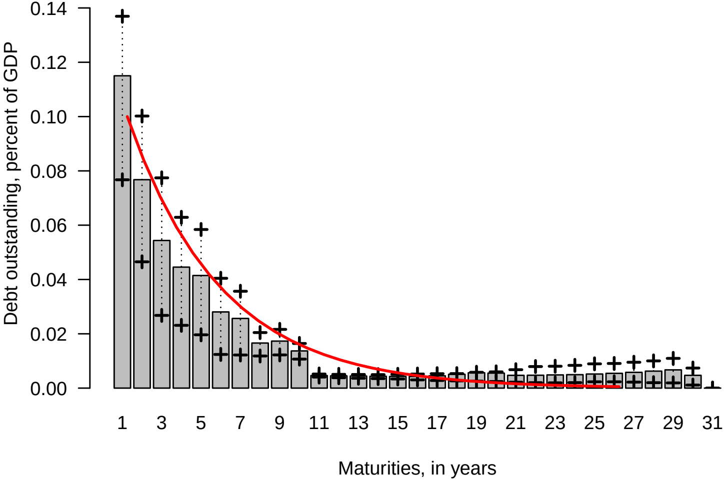
$$\Lambda_{t+h} = \underbrace{\chi^{h-1}}_{\text{decay}} \times \underbrace{[1 \times (1-\mathcal{D}_{t+h}) + RR \times \mathcal{D}_{t+h}]}_{\text{1 if no defaut and } RR \text{ otherwise}} \times \underbrace{\Pi_{t+h}^{\kappa_\pi} Y_{t+h}^{\kappa_y}}_{\text{composite index}}. \quad (1)$$

- $(\kappa_\pi = 0, \kappa_y = 0)$ = nominal bond; $(\kappa_\pi = 1, \kappa_y = 0)$ = TIPS; $(\kappa_\pi = 1, \kappa_y = 1)$ = GDP-indexed bond.
- Perpetuity price (in dollars): $\Pi_t^{\kappa_\pi} Y_t^{\kappa_y} \mathcal{P}_t = \sum_{i=1}^{\infty} \mathbb{E}_t(\mathcal{M}_{t,t+h}\Lambda_{t+h})$.
  Hence $\mathcal{P}_t$ = perpetuity price expressed in units of composite index.
- Perpetuity's yield-to-maturity q_t (internal rate of return) defined through:

$$\mathcal{P}_t = \sum_{h=1}^{\infty} \frac{\chi^{h-1}}{(1+q_t)^h} = \frac{1}{1+q_t-\chi}. \quad (2)$$



Note: Payoffs (in dollars) associated with a nominal perpetuity with geometrically-decaying coupons. Two values of χ (decay rate) are considered.



Note: Average redemption schedule (2000-2023). Crosses indicate 25th and 75th percentiles. The red line shows repayment schedule associated with a perpetuity featuring geometrically-decaying payoffs.

- In this context, standard dynamics of debt-to-GDP ratio d_t:

$$\boxed{d_{t+1} = \frac{1}{\pi_{t+1} + \Delta y_{t+1}} d_t - s_{t+1} + r_{t+1},}$$

where accounting convention = nominal valuation of debt securities.

- r_{t+1}, the debt service (in percent of GDP), comprises two components:

debt indexation + pre-indexation debt service ($\underline{r}_t$).

- Units of the two components depend on κ_π and κ_y. For instance:

Type of perpetuity	**Debt indexation** is homogeneous to:	**Pre-index. debt service** is homogeneous to:
$(\kappa_\pi = 0, \kappa_y = 0)$ = nominal	$-$	nominal rate
$(\kappa_\pi = 1, \kappa_y = 0)$ = TIPS	inflation	real rate
$(\kappa_\pi = 1, \kappa_y = 1)$ = GDP-L	nominal GDP growth	real rate "$-$" GDP growth

Proposition 1 - Debt dynamics

- In the absence of default until date $t+1$, we have:

$$
\begin{aligned}
d_{t+1} &= \exp(-\pi_{t+1} - \Delta y_{t+1}) d_t - s_{t+1} + r_{t+1} & (3)\\
r_{t+1} &= \underbrace{(\exp(\kappa_\pi \pi_{t+1} + \kappa_y \Delta y_{t+1}) - 1)\exp(-\pi_{t+1} - \Delta y_{t+1}) d_t}_{\text{debt indexation } (\zeta_{t+1} - \underline{\zeta}_{t+1}) d_t} + \underline{r}_{t+1} & (4)\\
\underline{r}_{t+1} &= \zeta_{t+1} q(z_t) \underbrace{(d_t - \chi \zeta_t d_{t-1})}_{\text{date-}t\text{ issuances}} + \zeta_{t+1} \chi \underline{r}_t, & (5)
\end{aligned}
$$

where

$$
\begin{cases}
\zeta_t = \exp[(\kappa_\pi - 1)\pi_t + (\kappa_y - 1)\Delta y_t] \\
\underline{\zeta}_t = \exp(-\pi_t - \Delta y_t).
\end{cases}
$$

- Flexible general specification of the stochastic discount factor (SDF):

$$\boxed{\mathcal{M}^r_{t,t+1} = \exp(f^r(z_t, z_{t+1}) + \nu^r(z_{t+1})\Delta\mathcal{D}_{t+1}),} \quad (6)$$

  where z_t is the state vector (includes d_t, r_t, π_t, Δy_t).

- **Empirical application**:
  Specification of f^r and ν^r based on Epstein-Zin preferences (Bansal and Yaron, 2004; Piazzesi and Schneider, 2007; Bansal and Shaliastovich, 2013).
  Only two parameters to calibrate: RRA and IES.

- Flexible general specification of the stochastic discount factor (SDF):

$$\boxed{\mathcal{M}^r_{t,t+1} = \exp(f^r(z_t, z_{t+1}) + \nu^r(z_{t+1})\Delta\mathcal{D}_{t+1}),} \quad (6)$$

where z_t is the state vector (includes d_t, r_t, π_t, Δy_t).

- **Empirical application**:
Specification of f^r and ν^r based on Epstein-Zin preferences (Bansal and Yaron, 2004; Piazzesi and Schneider, 2007; Bansal and Shaliastovich, 2013).
Only two parameters to calibrate: RRA and IES.

Proposition 2 - Perpetuity pricing

Function q satisfies the following fixed-point problem:

$$q(z_t) = \chi - 1 + \quad (7)$$
$$\frac{1}{\mathbb{E}_t\left(e^{f(z_t,z_{t+1})}\left[\mathcal{D}_{t+1}\left(RRe^{\nu^r(z_{t+1})}(1+\chi\underline{\mathcal{P}}(z_{t+1})) - \frac{1+q(z_{t+1})}{1+q(z_{t+1})-\chi}\right) + \frac{1+q(z_{t+1})}{1+q(z_{t+1})-\chi}\right]\right)},$$

where $\underline{\mathcal{P}}$ is the post-default price of the perpetuity, that is:

$$\underline{\mathcal{P}}(z_t) = \mathbb{E}\left(\sum_{h=1}^{\infty}\chi^{h-1}\mathcal{M}_{t,t+h}\middle|\mathcal{D}_t = 1, z_t\right).$$

Probability of default

- In the spirit of Pallara and Renne (2024), conditional probability of default:

$$\mathbb{P}(\mathcal{D}_{t+1} = 1 | \mathcal{D}_t = 0, z_t) = 1 - \exp(- \underbrace{\max[0, \alpha(d_t - d^*)]}_{=\lambda_{t+1}, \text{ default intensity}}), \tag{8}$$

where d^* = "fiscal limit".

$$\boxed{\Rightarrow PD_{t+1} \approx \max[0, \alpha(d_t - d^*)].}$$

- Large α (and $\nu_t = 0$) $\Rightarrow$ "strict" fiscal limit (default as soon as $d_t > d^*$).
- Small α (and $\nu_t = 0$) $\Rightarrow$ "soft" fiscal limit.

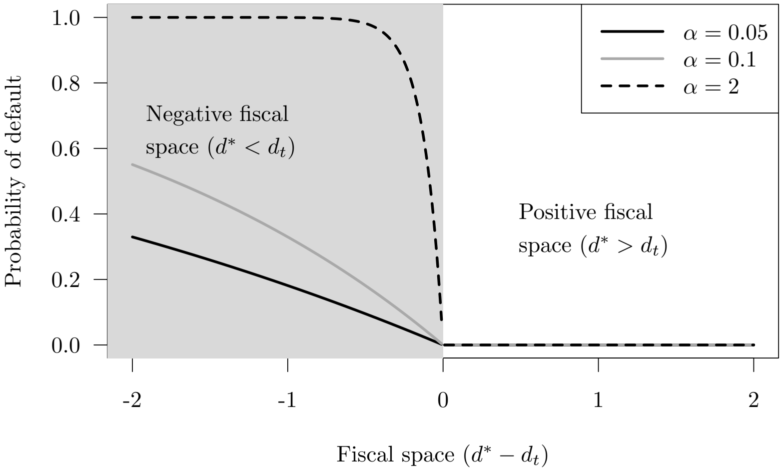


Figure: Probability of default as a function of fiscal space

Macro dynamics (and default feedback effects)

- Inflation and GDP growth:

$$\pi_t = \mu'_\pi m_t + \nu_\pi \Delta\mathcal{D}_t, \quad \text{and} \quad \Delta y_t = \mu'_y m_t + \nu_y \Delta\mathcal{D}_t, \tag{9}$$

where m_t is a selection vector of dimension $n_m \times 1$. Dynamics:

$$\mathbb{P}(m_{t+1} = e_j | m_t = e_i) = \Omega_{i,j},$$

Ω = matrix of transition probabilities. (e_j = j^{th} column of $m \times m$ identity matrix.)

- ν_π and ν_y capture feedback effects of sovereign default on inflation and GDP.
- Primary budget surplus:

$$s_t = s^* + \underbrace{\beta \times d_{t-1}}_{\text{stabilization component}} + \underbrace{\eta_t,}_{\text{risk component}} \tag{10}$$

where the term βd_{t-1} (with $\beta > 0$) = government's desire to stabilize the debt (as in, e.g., Bohn, 1998; Ghosh et al., 2013), and where:

$$\eta_t = \underbrace{\varepsilon_t}_{\mathcal{N}(0,\sigma^2)} + \underbrace{\mu'_\eta(m_t - \mathbb{E}_{t-1}(m_t)).}_{\text{links with macro innovations}}$$

Other bonds

- In this economy, one can price any other asset whose payoffs depend on z_t (even if not issued by the government).

Proposition 3

The prices of zero-coupon bonds can be computed recursively using:

$$\begin{aligned}\mathcal{B}_h(z_t) \quad = \quad & \mathbb{E}\Big[\exp(f(z_t, z_{t+1}))\mathcal{B}_{h-1}(z_{t+1}) + \\ & \mathcal{D}_{t+1}\exp(f(z_t, z_{t+1}))\left\{RRe^{\nu(z_{t+1})}\underline{\mathcal{B}}_{h-1}(z_{t+1}) - \mathcal{B}_{h-1}(z_{t+1})\right\}\Big|\,\mathcal{D}_t = 0, z_t\Big],\end{aligned} \tag{11}$$

starting from $\mathcal{B}_0(x) = 1$ for any state x. In (11), $\underline{\mathcal{B}}_h(z_t)$ denotes the price of a post-default zero-coupon bond, i.e.:

$$\underline{\mathcal{B}}_h(z_t) = \mathbb{E}(\mathcal{M}_{t,t+h}|\mathcal{D}_t = 1, z_t).$$

Cost and Risk measures

- **Average debt-to-GDP ratio and average debt service**
  Both criteria reflect the funding costs associated with the different strategies.
- **Debt volatility**
  Two measures: $\sqrt{\mathbb{V}ar(d_t)}$ and $\sqrt{\mathbb{V}ar(\Delta d_t)}$.
- **Upper tail of the debt-to-GDP distribution**
  95^{th} percentile of the debt-to-GDP distribution; characterizes right tail of the debt distribution.
- **Debt service volatility**
  Measured by $\sqrt{\mathbb{V}ar(r_t)}$.
- **Credit risk**
  Measured by the average 10-year probability of default. Formally: $\mathbb{E}(\mathbb{P}(\mathcal{D}_{t+10}|\mathcal{D}_t = 0, z_t))$.
- **Credit-risk costs**
  Measured by the average 10-year credit spread, that is formally given by $\mathbb{E}(y_{t,10} - y^*_{t,10})$, where $y_{t,10}$ is the govt 10-year nominal yield and $y^*_{t,10}$ is the yield of an equivalent non-defaultable bond.

Insights from stylized economies: demand/supply-driven

- Relative importance of demand/supply shocks shapes term structure of bond returns (e.g., Rudebusch and Swanson, 2012; Campbell et al., 2017; Bekaert et al., 2021).
- Two synthetic economies:

  Economy "D": demand shocks (+ correlation between inflat. and GDP growth).

  Economy "S": supply shocks.

Table: Stylized models: parameterizations

Regime	μ_π		μ_y	Ω		
	D	S				
1	0.000	0.060	0.000	0.800	0.200	0.000
2	0.030	0.030	0.020	0.100	0.800	0.100
3	0.060	0.000	0.040	0.000	0.200	0.800

Notes: This table shows the parameterizations of the stylized demand/supply models. We also have $\alpha = 0.10$, $\beta = 0.10$, $\gamma = 10$, $\beta = 0.10$, $d^* = 1.00$, $s^* = -0.08$, $\sigma_\nu = 0.10$, $\nu_\pi = 0.00$, $RR = 0.50$.

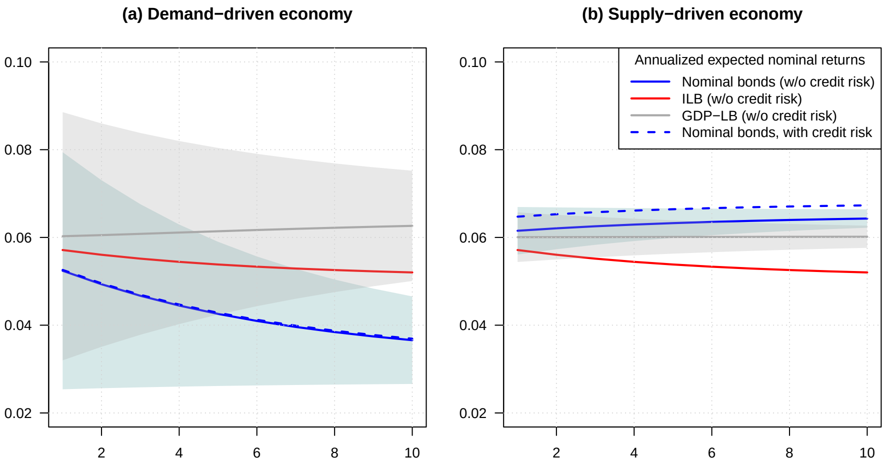


Figure: Term structures of expected returns in the two synthetic economies

Table: Performances of debt issuance strategies in stylized versions of the model, $\mu_\eta = 0 \times \mu_y$ and $\nu_y = 0$

	$\mathbb{E}(d)$	$\sqrt{\mathbb{V}(d)}$	$q_{95}(d)$	$\mathbb{E}(r)$	$\sqrt{\mathbb{V}(r)}$	$\sqrt{\mathbb{V}(\Delta d)}$	$\mathbb{E}(PD)$	$\mathbb{E}(spd)$
Coupon decay rate $\chi = 0.2$								
	Demand-driven economy ($\chi = 0.2$)							
Nominal	85.94	8.26	98.05	4.18	1.82	3.01	0.94	6.98
ILB	89.78	6.48	98.57	4.94	2.45	2.35	1.28	7.44
GDP-LB	94.58	5.55	99.97	5.65	3.14	2.38	2.30	11.59
	Supply-driven economy ($\chi = 0.2$)							
Nominal	97.18	7.49	107.40	6.08	0.77	2.39	3.58	17.18
ILB	89.79	6.50	98.59	5.04	1.42	2.35	1.28	7.46
GDP-LB	94.97	5.24	100.01	5.77	0.76	2.36	2.38	11.99
Coupon decay rate $\chi = 0.9$								
	Demand-driven economy ($\chi = 0.9$)							
Nominal	76.52	11.76	93.30	2.82	0.36	3.03	0.44	3.78
ILB	84.71	8.00	96.03	4.16	1.64	2.33	0.73	4.64
GDP-LB	94.38	5.94	100.95	5.68	3.21	2.38	2.31	11.56
	Supply-driven economy ($\chi = 0.9$)							
Nominal	100.96	8.18	112.55	6.63	0.65	2.28	5.38	25.31
ILB	85.25	8.48	98.14	4.36	1.95	2.33	0.90	5.68
GDP-LB	94.36	5.67	100.40	5.72	0.73	2.36	2.26	11.31

Notes: This table shows performance metrics associated with three different debt issuance strategies; each strategy consists in issuing a given type of perpetuities: a nominal perpetuity ($\kappa_\pi = 0$ and $\kappa_y = 0$), an inflation-indexed perpetuity nominal ($\kappa_\pi = 1$ and $\kappa_y = 0$), and a GDP-indexed perpetuity nominal ($\kappa_\pi = 1$ and $\kappa_y = 1$). We consider two different values of χ (the higher χ, the higher the average debt maturity). 'd' denotes the debt-to-GDP ratio. 'r' denotes the debt service, including debt indexation (in percent of GDP). '$\sqrt{\mathbb{V}(x)}$' corresponds to the standard deviation of variable x; 'PD' stands for '10-year probability of default' (expressed in percent); 'spd' stands for '10-year credit spread' (expressed in basis point), '$q_{95}(d)$' is the 95^{th} percentile of the debt-to-GDP distribution.

Model calibrated to US economy

- Some parameters taken from the literature. Examples:
  - ν_y set to -5% (Mendoza and Yue, 2012; Reinhart and Rogoff, 2011).
  - Coefficient of RRA set to 10 (Bansal and Yaron, 2004).
  - 0.5 elasticity of surplus to output, i.e., $\mu_\eta = 0.5 \times \mu_y$ (van den Noord, 2000).
- Core step of the calibration process: Π, μ_π, and μ_z.
  Equilibrium model $\Rightarrow$ critical importance of macro dynamics to shape YC.
  Estimation approach combines fit of *fluctuations* and *average* values of yields.

$\Rightarrow$ Denoting the vector of parameters to be estimated by Θ:

$$\hat{\Theta} = \underset{\Theta}{\text{argmin}} - \log \mathcal{L}(\Theta) + d(\Theta),$$

where $\log \mathcal{L}(\Theta)$ is the log-likelihood function and $d(\Theta)$ is a measure of the distance between model-implied and targeted yield moments.

- 5 regimes. Estimation period: 1970-2023.

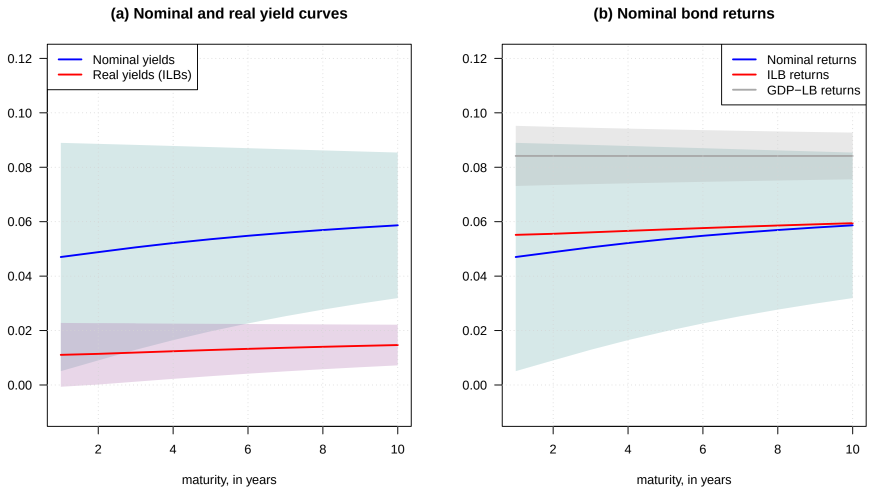
param. table fit of yield time series

Table: Model-implied versus targeted moments

Moment	Model	Target
Avg. slope of nominal yield curve (1y-10y)	0.012	0.011
Avg. 10-year nominal yield	0.059	0.060
Avg. slope of real yield curve (2y-10y)	0.003	0.009
Avg. 10-year real yield	0.015	0.014
Avg. inflation	0.044	0.039
Avg. real GDP growth	0.029	0.027
Std dev. of 10-year nominal yield	0.027	0.030
Std dev. of 10-year real yield	0.007	0.013
Avg. breakeven	0.000	0.000

Notes: This table compares model-implied with targeted moments. The distance between these moments is part of the loss function that is minimized to estimate the components of μ_π, μ_y, and Ω. See Subsection **??** for more details.

Model-implied yield curves and returns

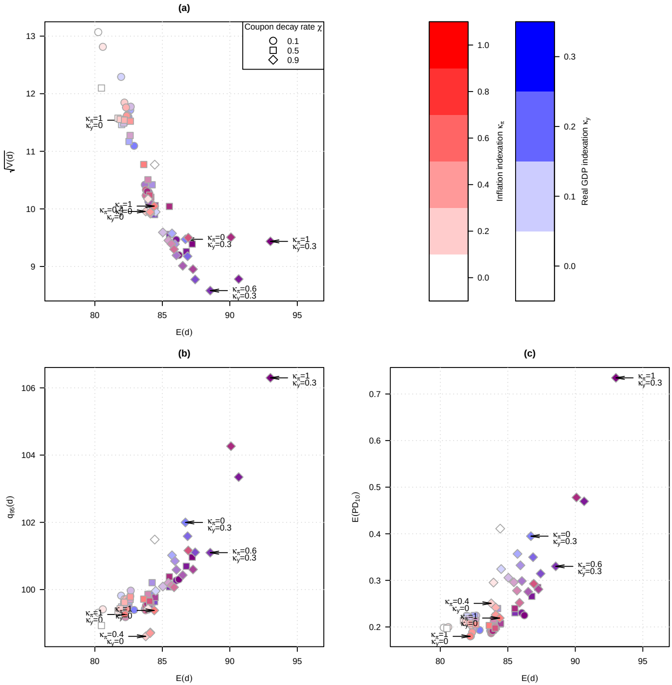


Model-implied performances of selected strategies

Table: Performances of debt issuance strategies in the calibrated model

$(\chi,\kappa_\pi,\kappa_y)$	$\mathbb{E}(d)$	$\sqrt{\mathbb{V}(d)}$	$q_{95}(d)$	$\mathbb{E}(r)$	$\sqrt{\mathbb{V}(r)}$	$\sqrt{\mathbb{V}(\Delta d)}$	$\mathbb{E}(PD)$	$\mathbb{E}(spd)$
$(0.1, 0.0, 0.0)$	80.25	13.07	99.31	3.97	3.73	7.52	0.20	9.08
$(0.1, 0.0, 0.3)$	82.91	11.09	99.39	4.75	3.22	6.98	0.19	5.94
$(0.1, 1.0, 0.0)$	82.22	11.54	99.26	4.70	3.36	6.96	0.18	4.68
$(0.1, 1.0, 0.3)$	86.22	9.20	100.30	5.69	2.80	6.18	0.22	4.30
$(0.9, 0.0, 0.0)$	84.44	10.77	101.49	4.97	1.91	8.51	0.41	23.03
$(0.9, 0.0, 0.3)$	86.71	9.47	102.00	5.57	1.85	7.42	0.39	17.14
$(0.9, 1.0, 0.0)$	84.40	10.05	99.38	5.04	2.68	7.21	0.22	7.66
$(0.9, 1.0, 0.3)$	93.01	9.44	106.30	7.41	2.99	6.10	0.73	16.87
$(0.9, 0.6, 0.3)$	88.55	8.58	101.10	6.10	2.23	6.46	0.33	10.87
$(0.9, 0.4, 0.0)$	83.77	9.96	98.61	4.90	2.09	7.49	0.25	13.49
$(0.1, 1.0, 0.0)$	82.22	11.54	99.26	4.70	3.36	6.96	0.18	4.68

Notes: This table shows performance metrics associated with different debt issuance strategies characterized by the issuance of perpetuities of different durations (captured by the coupon decay rate χ), a coefficient of indexation to inflation κ_π and a coefficient of indexation to real GDP κ_y. The model is the one whose parameterization is reported in Table 5. 'd' denotes the debt-to-GDP ratio. 'r' denotes the debt service, including debt indexation (in percent of GDP). '$\sqrt{\mathbb{V}(x)}$' corresponds to the standard deviation of variable x; 'PD' stands for '10-year probability of default' (expressed in percent); 'spd' stands for '10-year credit spread' (expressed in basis point), '$q_{95}(d)$' is the 95^{th} percentile of the debt-to-GDP distribution. The last three rows show the performances of the strategies implying the lowest $\sqrt{\mathbb{V}(d)}$, $q_{95}(d)$, and $\mathbb{E}(PD)$, respectively.



(a)
Coupon decay rate χ
0.1
0.5
0.9
√V(d)
E(d)
κπ=1 κy=0
κπ=1 κy=0
κπ=0.4 κy=0
κπ=0 κy=0.3
κπ=1 κy=0.3
κπ=0.6 κy=0.3
Inflation indexation κπ
Real GDP indexation κy
(b)
q95(d)
E(d)
(c)
E(PD10)
E(d)

Concluding remarks

- This paper proposes a framework to analyze public debt management.
- Stochastic macro-finance equilibrium model where govt decides on maturity and debt indexation. Moreover, govt faces fiscal limit.
- Endogenous bond prices.
- Quasi analytical solutions; no need for Monte-Carlo simulations.
- Issuance Cost/risk performances of issuance strategies.
- Empirical application on U.S. data.
- Replication package (in R) available at https://github.com/jrenne/PDMAnalyt.

Concluding remarks

- This paper proposes a framework to analyze public debt management.
- Stochastic macro-finance equilibrium model where govt decides on maturity and debt indexation. Moreover, govt faces fiscal limit.
- Endogenous bond prices.
- Quasi analytical solutions; no need for Monte-Carlo simulations.
- Issuance Cost/risk performances of issuance strategies.
- Empirical application on U.S. data.
- Replication package (in R) available at https://github.com/jrenne/PDMAnalyt.

Thanks!

References I

Angeletos, G.-M. (2002). Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure. *The Quarterly Journal of Economics*, 117(3):1105–1131.

Balibek, E. and Memis, H. A. (2012). Turkish Treasury Simulation Model for Debt Strategy Analysis. Policy Research Working Paper Series 6091, The World Bank.

Bansal, R. and Shaliastovich, I. (2013). A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets. *Review of Financial Studies*, 26(1):1–33.

Bansal, R. and Yaron, A. (2004). Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance*, 59:1481–1509.

Barro, R. (2003). Optimal Management of Indexed and Nominal Debt. CEMA Working Papers 496, China Economics and Management Academy, Central University of Finance and Economics.

Barro, R. J. (1979). On the Determination of the Public Debt. *Journal of Political Economy*, 87(5):940–971.

Barro, R. J. (1995). Optimal Debt Management. Working Paper 5327, National Bureau of Economic Research.

Bekaert, G., Engstrom, E., and Ermolov, A. (2021). Macro Risks and the Term Structure of Interest Rates. *Journal of Financial Economics*, 141(2):479–504.

Bergstrom, P., Holmlund, A., and Lindberg, S. (2002). The SNDO's Simulation Model for Government Debt Analysis. SNDO Working Paper, Swedish National Debt Office.

Bernaschi, M., Morea, R., Sarno, L., Tesseri, F., Verani, F., and Vergni, D. (2019). An Integrated Approach to Cost-Risk Analysis in Public Debt . Technical report, Dipartemento del Tesoro.

Bhandari, A., Evans, D., Golosov, M., and Sargent, T. J. (2017). Fiscal Policy and Debt Management with Incomplete Markets. *The Quarterly Journal of Economics*, 132(2):617–663.

Bigio, S., Nuño, G., and Passadore, J. (2023). Debt-Maturity Management with Liquidity Costs. *Journal of Political Economy Macroeconomics*, 1(1):119–190.

Bocola, L. and Dovis, A. (2019). Self-Fulfilling Debt Crises: A Quantitative Analysis. *American Economic Review*, 109(12):4343–77.

Bohn, H. (1990). Tax Smoothing with Financial Instruments. *The American Economic Review*, 80(5):1217–1230.

Bohn, H. (1998). The Behavior of U.S. Public Debt and Deficits. *Quarterly Journal of Economics*, 113(3):949–963.

References II

Bolder, D. J. and Deeley, S. (2011). The Canadian Debt-Strategy Model: An Overview of the Principal Elements. Bank of Canada Staff Discussion Paper 2011-03, Bank of Canada.

Borensztein, E. and Mauro, P. (2004). The Case for GDP-Indexed Bonds. *Economic Policy*, 19(38):165–216.

Buera, F. and Nicolini, J. P. (2004). Optimal Maturity of Government Debt without State Contingent Bonds. *Journal of Monetary Economics*, 51(3):531–554.

Campbell, J. Y., Sunderam, A., and Viceira, L. M. (2017). Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds. *Critical Finance Review*, 6(2):263–301.

Cole, H. L. and Kehoe, T. J. (2000). Self-Fulfilling Debt Crises. *The Review of Economic Studies*, 67(1):91–116.

de Lannoy, L. R. A., Bhandari, A., Evans, D., Golosov, M., and Sargent, T. J. (2022). Managing Public Portfolios. NBER Working Papers 30501, National Bureau of Economic Research, Inc.

Debortoli, D., Nunes, R., and Yared, P. (2017). Optimal Time-Consistent Government Debt Maturity. *The Quarterly Journal of Economics*, 132(1):55–102.

Drudi, F. and Giordano, R. (2000). Default Risk and Optimal Debt Management. *Journal of Banking & Finance*, 24(6):861–891.

Faraglia, E., Marcet, A., and Scott, A. (2010). In Search of a Theory of Debt Management. *Journal of Monetary Economics*, 57(7):821–836.

Froot, K. A., Scharfstein, D. S., and Stein, J. C. (1989). LDC Debt: Forgiveness, Indexation, and Investment Incentives. *The Journal of Finance*, 44(5):1335–1350.

Ghosh, A., Kim, J. I., Mendoza, E., Ostry, J., and Qureshi, M. (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *The Economic Journal*, 123(566):F4–F30.

Greenwood, R., Hanson, S. G., and Stein, J. C. (2015). A Comparative-Advantage Approach to Government Debt Maturity. *The Journal of Finance*, 70(4):1683–1722.

Hatchondo, J. C. and Martinez, L. (2009). Long-Duration Bonds and Sovereign Defaults. *Journal of International Economics*, 79(1):117–125.

References III

Kamstra, M. J. and Shiller, R. J. (2009). The Case for Trills: Giving the People and Their Pension Funds a Stake in the Wealth of the Nation. Cowles Foundation Discussion Papers 1717, Cowles Foundation for Research in Economics, Yale University.

Kamstra, M. J. and Shiller, R. J. (2010). Trills Instead of T-Bills: It's Time to Replace Part of Government Debt with Shares in GDP. *The Economists' Voice*, 7(3).

Leland, H. (1998). Agency Costs, Risk Management, and Capital Structure. *Journal of Finance*, 53:1213–1243.

Mendoza, E. G. and Yue, V. Z. (2012). A General Equilibrium Model of Sovereign Default and Business Cycles. *Quarterly Journal of Economics*, 127(2):889–946.

Missale, A. (1997). Managing the Public Debt: The Optimal Taxation Approach. *Journal of Economic Surveys*, 11(3):235–265.

Missale, A. (2012). Sovereign Debt Management and Fiscal Vulnerabilities. In for International Settlements, B., editor, *Threat of Fiscal Dominance?*, volume 65 of *BIS Papers chapters*, pages 157–176. Bank for International Settlements.

Missale, A. and Blanchard, O. J. (1994). The Debt Burden and Debt Maturity. *The American Economic Review*, 84(1):309–319.

Missale, A. and Giavazzi, F. (2005). Public Debt Management in Brazil. In *Inflation Targeting, Debt, and the Brazilian Experience, 1999 to 2003*. The MIT Press.

Pallara, K. and Renne, J.-P. (2024). Fiscal Limits and the Pricing of Eurobonds. *Management Science*, 70(2):1216–1237.

Piazzesi, M. and Schneider, M. (2007). Equilibrium Yield Curves. In *NBER Macroeconomics Annual 2006, Volume 21*, NBER Chapters, pages 389–472. National Bureau of Economic Research.

Pick, A. and Anthony, M. (2008). A Simulation Model for the Analysis of the UK's Sovereign Debt Strategy. UK DMO Working Paper, UK DMO.

Pienkowski, A. (2017). Debt Limits and the Structure of Public Debt. IMF Working Papers 17/117, International Monetary Fund.

Reinhart, C. M. and Rogoff, K. S. (2011). The Forgotten History of Domestic Debt. *Economic Journal*, 121(552):319–350.

Rudebusch, G. D. and Swanson, E. T. (2012). The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks. *American Economic Journal: Macroeconomics*, 4(1):105–143.

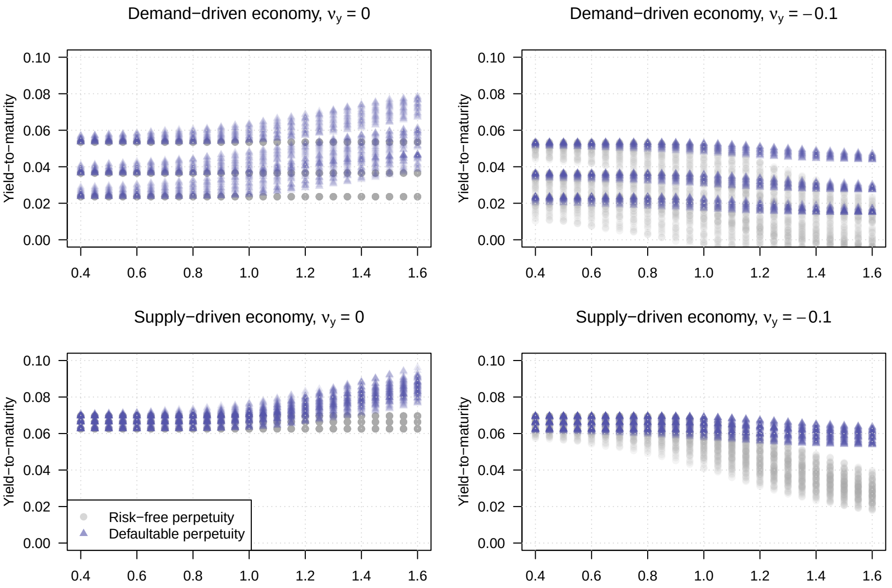
References IV

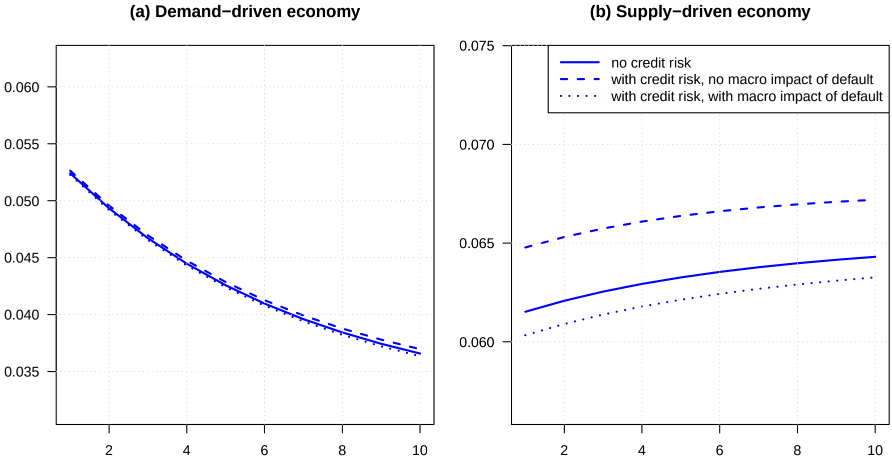
Schmid, L., Valaitis, V., and Villa, A. (2023). Government Debt Management and Inflation with Real and Nominal Bonds. CEPR Discussion Papers 18197, C.E.P.R. Discussion Papers.

Shiller, R. J. (1998). *Macro Markets: Creating Institutions for Managing Society's Largest Economic Risks*. Number 9780198294184 in OUP Catalogue. Oxford University Press.

van den Noord, P. (2000). The Size and Role of Automatic Fiscal Stabilizers in the 1990s and Beyond. OECD Economics Department Working Papers 230, OECD.

Woodford, M. (2001). Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking*, 33(3):669–728.


Demand−driven economy, $\nu_y = 0$
Demand−driven economy, $\nu_y = -0.1$
Supply−driven economy, $\nu_y = 0$
Supply−driven economy, $\nu_y = -0.1$
Yield−to−maturity
0.00
0.02
0.04
0.06
0.08
0.10
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Risk−free perpetuity
Defaultable perpetuity


(a) Demand−driven economy
(b) Supply−driven economy
0.060
0.055
0.050
0.045
0.040
0.035
2
4
6
8
10
0.075
0.070
0.065
0.060
no credit risk
with credit risk, no macro impact of default
with credit risk, with macro impact of default

Model parameterization – U.S. economy back

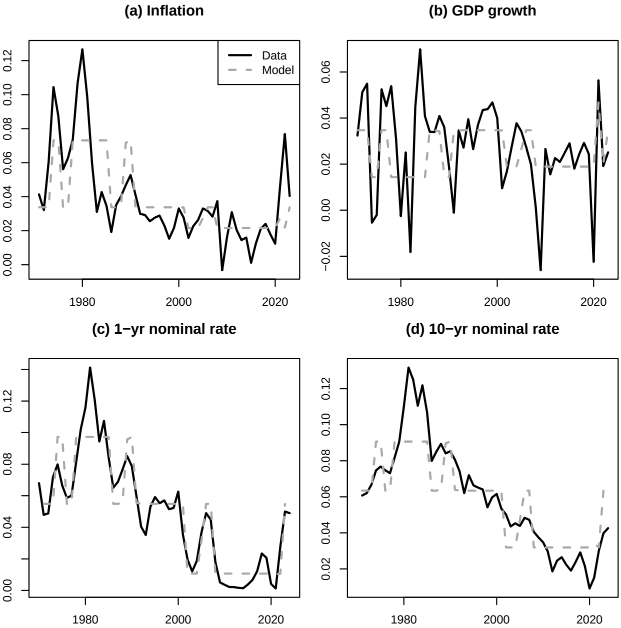
Table: Model parameterization

Regime	μ_π	μ_y			Ω		
1	0.030	0.060	0.867	0.133	0.000	0.000	0.000
2	-0.016	-0.100	0.715	0.118	0.167	0.000	0.000
3	0.073	0.014	0.029	0.000	0.962	0.008	0.000
4	0.034	0.035	0.000	0.063	0.275	0.634	0.028
5	0.022	0.019	0.001	0.196	0.000	0.051	0.752

Notes: This table shows the model parameterization of the baseline model. We also have: $\alpha = 0.1$, $\beta = 0.20$, $\gamma = 10$, $\delta = 0.99$, $d^* = 1.10$, $s^* = -0.176$, $\nu_y = -0.050$, $\nu_\pi = -0.021$, $\mu_\eta = 0.5 \times \mu_y$, $RR = 0.50$.

Fitted yields – U.S. economy

back



Jean-Paul Renne

An Analytical Framework for Public Debt Management

3rd PDM conference, World Bank.